

**NC STATE UNIVERSITY**

**College of Engineering**

**Department of Mechanical and Aerospace Engineering**



MAE-305, Section 205

Mechanical Engineering Laboratory I – Instrumentation & Solid Mechanics  
Laboratory

Experiment #5

**Stress and Strain Measurement Using Digital  
Image Correlation**

**Written By: David Delgado**  
**Lab Partner: Hannah Fletcher**  
**Lab Partner: Sunghyun Shin**  
**Lab Partner: Leena Vo**

**Date Experiment Performed: 10/25/2018**  
**Date Report Submitted: 11/01/2018**

## Abstract

[Primary Contributor: David Delgado]

This experiment determined the linearity of strain along a cantilevered beam's axis using a 3D optical strain measurement technique called Digital Image Correlation (DIC). An aluminum beam, with a known modulus of elasticity, was installed into a flexor setup with two cameras pointed at the beam to measure 3D displacements by capturing images of the surface. From these displacements, VIC-3D was used to calculate strain measurements at three locations along the beam.

The results were plotted on a strain versus distance graph and their linearity verified. Then, stress for three different locations was calculated using two equations: one as a function of external load (Equation 1) and the other as a function of strain (Equation 2).

$$\sigma_i = \frac{6Px_i}{bt^2} \text{ [psi]} \quad i = 1,2,3 \quad (1)$$

$$\sigma_i = E\varepsilon_i \text{ [psi]} \quad i = 1,2,3 \quad (2)$$

Then, the percent difference for the stress values at all three locations was calculated using the following:

$$\% \text{ diff} = \frac{|Value_{exp} - Value_{pub}|}{\frac{1}{2}(Value_{exp} + Value_{pub})} \times 100\% \quad (3)$$

It was found that, while the results were linear, only one location provided decent accuracy (15.4%) in stress measurements. New images would need to be captured and equipment calibrated to ensure data is collected properly, and the experiment repeated in order to obtain a higher level of accuracy in the data.

## 1. Introduction

[Primary Contributor: David Delgado]

The purpose of this experiment was to determine the linearity of strain along a cantilevered beam's axis using a 3D optical strain measurement technique called Digital Image Correlation (DIC). DIC is an optical method (speckle tracking) that tracks the movement of random dot patterns on the specimen surface to calculate 2D and 3D displacements. From these displacements it calculates additional quantities like strain, velocity, and acceleration. This optical approach is widely applied in many areas of science and engineering, including medical imaging and fluid flow tracking. The method involves:

1. Applying a random dot pattern on the surface of the material.
2. Using a camera to capture images of the surface.
3. Using a computer algorithm (i.e., DIC) to track the movement of the dots and calculate surface displacements.

This particular experiment built off the knowledge gained last week using strain gages to acquire similar strain measurements, as well as the knowledge gained in previous labs regarding instrumentation systems and beam mechanics. The significance of the VIC-3D system introduced in this experiment lies in the fact that it can replace traditional strain gages in cases where it is not practical to bond strain gages to the surface. Both approaches provide estimated displacements for a surface, which is useful in finding localization or damage that occurs in a material.

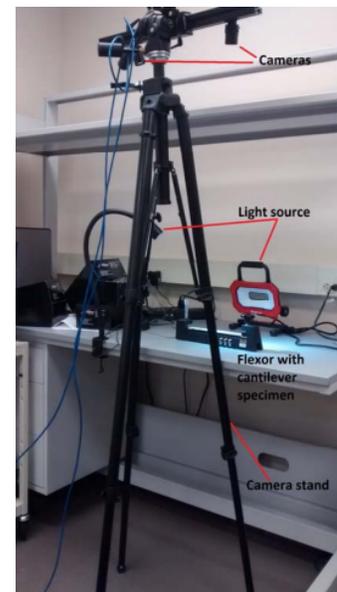
## 2. Experimental Methods

[Primary Contributor: David Delgado]

In this experiment the strain on the top surface of a cantilever beam subject to bending moments was measured using the DIC technique.

Equipment needed (see Figure 2.1):

- 0.013" dot sized pattern
- Inkpad
- White spray paint
- 12" calibration grid
- Aluminum specimen
- Flexor (Vishay Measurements Group – S/N 136349)
- Fine and coarse sandpaper
- CSM-2 degreaser
- Two (2) cameras
- Camera stand
- Light source
- Computer
- VIC-3D software



**Figure 2.1.**  
**Experimental setup per**  
**the lab handout**

The accuracy of this experiment relies upon the calibration of both cameras prior to taking measurements. Setup is simple, requiring only a quick, flexible calibration procedure per the lab handout (Section 3.2), and an applied random speckle pattern. Once setup and calibration of the system is complete:

1. Place the aluminum specimen (with the speckled surface up) into the flexor
2. Capture an image in the unloaded condition. This will be the reference image
3. Turn the dial in small increments and at each increment capture a new image
4. Keep turning the dial until 40-50 images have been captured
5. Close VIC Snap
6. Analyse the images saved on the desktop using VIC-3D per the lab handout (Section 3.3)
7. Observe the values of strain obtained and compare to the values obtained using strain gages
  - a. Plot the strains versus the distance from the concentrated load in excel
  - b. Draw a straight, best-fit line through these data points
  - c. Calculate the maximum deviation of data point from the line.
  - d. Estimate the slope of the previous plot and use this value to calculate external load
  - e. Calculate stress for all three locations using two equations: one is a function of external load and the other is a function of strain.
  - f. Determine the percent difference between these two methods for the analysis.

### 3. Experimental Data

[Primary Contributor: David Delgado]

The modulus of elasticity given was:

- $E = 10.4 \times 10^6$  psi

The beam dimensions measured were:

- $b = 0.925$  in. (width)
- $t = 0.188$  in. (thickness)
- $l = 12$  in. (length)

The locations of the three test points from the point of load application were:

- $x_1 = 1$  in.
- $x_2 = 4$  in.
- $x_3 = 7$  in.

The individual strain measurements at the corresponding locations from the DIC were (see Figure 3.1):

- $\epsilon_1 = 131 \mu\epsilon$
- $\epsilon_2 = 513 \mu\epsilon$
- $\epsilon_3 = 817 \mu\epsilon$

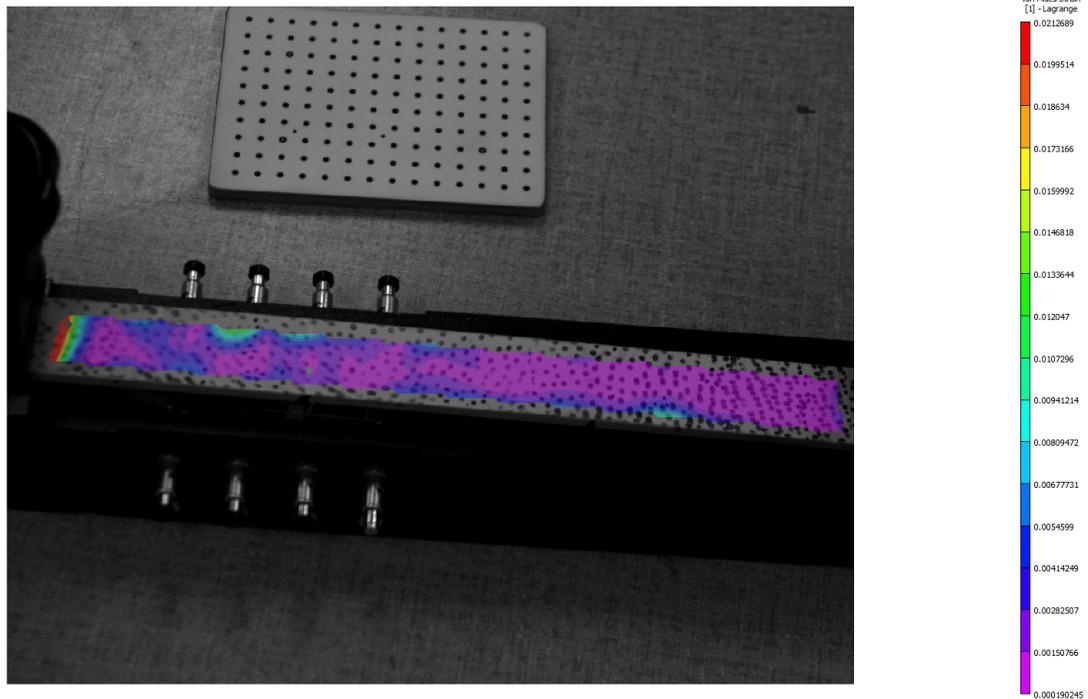


Figure 3.1. 2D strain contour generated by VIC-3D displaying optical strain measurements

#### 4. Theory and Analysis

[Primary Contributor: David Delgado]

Using the strain measurements given by the VIC-3D system, as well as the location at each point that strain was recorded, a graph of strain ( $\epsilon$ ) versus distance ( $x$ ) was created:

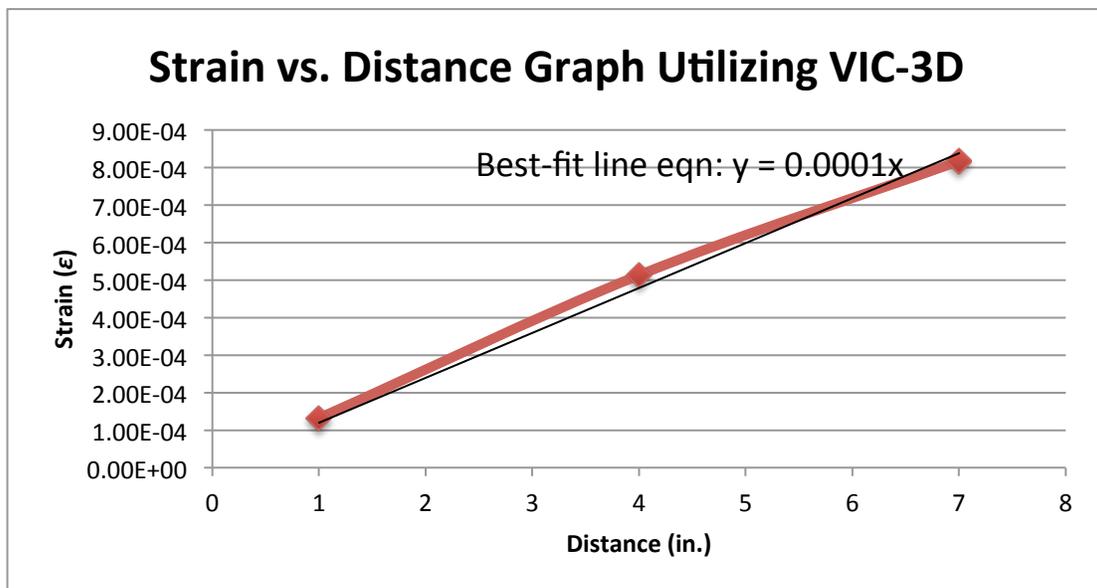


Figure 4.1. Strain vs. Distance from Fixed End for an Aluminum Beam

The slope of the strain versus distance plot was used to calculate the external load applied at the free end of the cantilever beam as such:

$$P = \frac{Ebt^2}{6} \times \frac{\Delta\varepsilon}{\Delta x} \text{ [lbs]} \quad (1)$$

Where  $E$  is the modulus of elasticity of the aluminum specimen,  $b$  is the width of the beam,  $t$  is the thickness of the beam and  $\frac{\Delta\varepsilon}{\Delta x}$  is the slope of the plot. The experimental value of this slope was defined as:

$$P = \frac{(10.4 \times 10^6 \text{ psi})(0.925 \text{ in})(0.188 \text{ in})^2}{6} \times (1 \times 10^{-4} \text{ } \varepsilon/\text{in}) = 5.6668 \text{ lbs} \quad (2)$$

Next, the stress at all three locations was calculated and entered into Table 4.1 as a function of the external load and as a function of strain using equations 3 and 4, respectively:

$$\sigma_i = \frac{6Px_i}{bt^2} \text{ [psi]} \quad i = 1,2,3 \quad (3)$$

$$\sigma_i = E\varepsilon_i \text{ [psi]} \quad i = 1,2,3 \quad (4)$$

Then, the percent difference between the two methods of calculating stress (i.e., Equations 3 and 4) was calculated using the following:

$$\% \text{ diff} = \frac{|Value_{exp} - Value_{pub}|}{\frac{1}{2}(Value_{exp} + Value_{pub})} \times 100\% \quad (5)$$

See Sample Calculations for a more thorough review of the experimental values used to arrive at the data shown in Table 4.1.

**Table 4.1. Stress for three locations as a function of external load and strain**

Distance from fixed end (in.)	Stress as a function of external load (psi)	Stress as a function of strain (psi)	% Difference
1	1040 psi	1360 psi	26.8 %
4	4160 psi	5340 psi	24.8 %
7	7280 psi	8500 psi	15.4 %

Lastly, the maximum deviation from the best-fit, straight line was measured from the graph:

$$\text{Maximum deviation} = 117 \mu\epsilon.$$

## 5. Discussion

[Primary Contributor: David Delgado]

The equations used to estimate displacement of the beam based on the VIC-3D system reveal that accuracy is largely dependent on not only proper calibration of the system, but also the precision of excel in computing the slope of the strain versus distance plot. The data from Table 4.1 shows that the percent error between the two methods of calculating stress decreases with distance from the fixed end of the cantilever beam. Thus, the strain was more accurately recorded at the furthest location from the fixed end. Specifically, the percent error between the two stress measurements at station 3 was 15.4%.

The results also show that strain increases linearly with distance along the beam. From equation 3 and equation 4, the logical projection is that stress along the cantilever beam also shares a linear relationship with distance along the beam axis. Nevertheless, the calculations of percent error reveal that the relationship is not perfectly linear, as the stress at the same location would need to produce the same value for both equation 3 and equation 4 for this to be the case. The reason that this perfectly linear relationship cannot be established is due to the optical system being used in this experiment. It is important to note that sources of error within this system include:

- Improper calibration of the cameras
- Inaccuracy in reading strain measurements from analysis tool (Figure 3.1)

In order to improve the accuracy of the analysis and better approximate the linearity of the strain, one could collect additional data points for each location and/or collect data at additional locations along the beam axis. The experiment could also benefit from repeating the experiment with the addition of strain gages as seen in Lab 4 to compare the strain measurements from two different methods. The accuracy of both procedures could then be estimated utilizing equation 5.

## 6. Conclusions

[Primary Contributor: David Delgado]

The purpose of this lab was to provide knowledge regarding stress and strain along a cantilever beam axis. In analyzing the data (especially the percent differences), it can be concluded that the gage at station 3 was providing the most accurate readings, while the gage at station 1 and 2 was off. Regardless, the data is mostly linear, indicating that the test was carried out as accurately as possible.

The first station's strain readings are a major source of error in the analysis. If one were to redo this experiment, the accuracy of the results would benefit from ensuring the VIC-3D optical strain measurement system is properly calibrated. Furthermore, more images could be acquired and analyzed after calibration in the form of separate trials. Then, the results of these separate trials could be averaged in the analysis, and a plot with three different stress versus distance curves generated (as well as one average curve). With at least three data readings at each loading, it would be very easy to see the linearity of strain along the beam axis.

## 7. References

[Primary Contributor: David Delgado]

- 1) Correlated solutions “VIC-3D reference manual”.
- 2) Lab Handout. MAE 305 Experiment 5: Strain measurement using digital image correlation.
- 3) Michael A Sutton, Jean-Jose Ardu, Hubert W. Schreier “Image Correlation for Shape, Motion and Deformation Measurements”.
- 4) VIC-3D User Interface.

## 8. Sample Calculations

[Primary Contributor: David Delgado]

Several equations were used to properly conduct the lab.

Let the slope of the strain versus distance plot be described by:

$$\frac{\Delta\varepsilon}{\Delta x} [\mu\varepsilon/in] \quad (1)$$

Where  $\Delta\varepsilon$  is the change in strain and  $\Delta x$  is the change in distance from the fixed end of the cantilever beam. The experimental value of this slope was defined as:

$$\frac{\Delta\varepsilon}{\Delta x} = 1 \times 10^{-4} \frac{\varepsilon}{in.} \quad (2)$$

Note that this result is valid only if the linearity of the data points is assumed in excel using a linear, best-fit curve. The result is units of microstrain per inches.

The external load applied at the free end of the cantilever beam was calculated using the slope defined above as such:

$$P = \frac{Ebt^2}{6} \times \frac{\Delta\varepsilon}{\Delta x} [lbs] \quad (3)$$

Where  $E$  is the modulus of elasticity of the aluminum specimen,  $b$  is the width of the beam and  $t$  is the thickness of the beam. The experimental value of this slope was defined as:

$$P = \frac{(10.4 \times 10^6 \text{psi})(0.925 \text{in})(0.188 \text{in})^2}{6} \times \left(1 \times 10^{-4} \frac{\varepsilon}{in.}\right) = 5.6668 \text{ lbs} \quad (4)$$

Note that this result relies heavily on accuracy of the thickness measurement and the slope of the curve calculated in Equation 2. In calculating  $P$ , the slope must be converted to units of strain per inches in order to have the final result be dimensionally correct.

The equation used to calculate stress as a function of the external load is as follows:

$$\sigma_i = \frac{6Px_i}{bt^2} \text{ [psi]} \quad i = 1,2,3 \quad (5)$$

Where  $P$  is the external load calculated in Equation 4,  $x$  is the distance from the fixed end of the cantilever beam,  $b$  is the width of the beam and  $t$  is the thickness of the beam. The experimental value of the stress at point 1 was defined as:

$$\sigma_1 = \frac{6(5.6668\text{lbs})(1\text{in})}{(0.925\text{in})(0.188\text{in})^2} = 1040 \text{ psi} \quad (6)$$

In finding the stress as a function of strain, the following equation was used:

$$\sigma_i = E\varepsilon_i \text{ [psi]} \quad i = 1,2,3 \quad (7)$$

Where  $E$  is the modulus of elasticity of the aluminum specimen and  $\varepsilon_i$  is the strain measurement (in microstrain) for each individual location on the beam. The experimental value of the stress at point 1 was defined as:

$$\sigma_1 = (10.4 \times 10^6 \text{psi})(131 \times 10^{-6} \varepsilon) = 1362.4 \text{ psi} \quad (8)$$

The percent difference between the two methods of calculating stress (i.e., Equation 5 and Equation 7) was calculated using the following:

$$\% \text{ diff} = \frac{|Value_{exp} - Value_{pub}|}{\frac{1}{2}(Value_{exp} + Value_{pub})} \times 100\% \quad (9)$$

Where the experimental value ( $Value_{exp}$ ) was found using Equation 5 and the published value ( $Value_{pub}$ ) was found using Equation 7. The percent difference at point 1 was defined as:

$$\% \text{ diff} = \frac{|1040\text{psi} - 1362.4\text{psi}|}{\frac{1}{2}(1040\text{psi} + 1362.4\text{psi})} \times 100\% = 26.840\% \quad (10)$$