NC STATE UNIVERSITY

College of Engineering

Department of Mechanical and Aerospace Engineering

MAE-305, Section 205

Mechanical Engineering Laboratory I – Instrumentation & Solid Mechanics Laboratory

Experiment #6

Stress in a Pressure Vessel

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Abstract

[Primary Contributor: David Delgado]

The objective of this experiment was to determine the average internal pressure of a pressurized beverage container at room temperature using experimentally collected circumferential and longitudinal strain measurements. From the six (6) pressure measurements calculated, the mean value of the data was found and the uncertainty of this pressure was determined using Kline McClintock's method and outliers were omitted to standardize the dataset via Chauvenet's Criterion. The mean pressure was determined to be x_m = - 58.495 psi with an uncertainty of 44.954% for hoop strain measurements and 87.127% for longitudinal strain measurements. By examining the Kline McClintock's equations for both of these methods, it was found that the thickness measurement was the greatest factor affecting the uncertainty of the pressure calculations. The latter may be due to ineffective micrometer measurements of the thickness and radius. Lastly, Chauvenet's Criterion allowed for two internal pressure values to be omitted from the analysis in order to achieve lower uncertainty values and better approximate the average pressure of the soda cans. The two values omitted were -26.300 psi and -107.06 psi.

1. Introduction

[Primary Contributor: David Delgado]

The purpose of this experiment was to study the deformation of a thin wall cylindrical structure due to internal pressure. Strain values were collected for several cans using strain-measuring methods learned in lab 4 of the course. From the collected data, a relationship between the strain of the soda can and its diameter, thickness and brand was established by applying statistical data analyses to the measurements. Knowing the procedure for calculating internal pressure and strain for cylindrical vessels is important as these devices are commonly used in the science and engineering field. Examples of these devices include air compressor tanks or storage containers used for transporting fluids. In summary, the tasks associated with this lab included:

1. Applying a strain gage to soda can and connecting it to the P3 strain measurement unit.
2. Collecting strain measurements while pressure in the can is relieved (by opening the can).
3. Conducting statistical analysis on the lab data.

This particular experiment built off the knowledge gained during lab 4 using strain gages to acquire similar strain measurements, as well as the knowledge gained in previous labs and solid mechanics regarding instrumentation systems and cylindrical stresses.

2. Experimental Methods

[Primary Contributor: David Delgado]

In this experiment the strain gage was set up to measure either longitudinal (i.e., axial) or hoop stress. Groups made a note of the application method, but first the proper equipment was gathered.

Equipment needed (see Figure 2.1):

- Unopened soda can
- 120 Ω strain gage (CEA-13-240UZ-120)
- P-3 strain measuring device (Vishay Micro-Measurements 136349)
- Soldering iron
- Solder
- Wire strippers
- M-Prep Conditioner A
- M-Prep Neutralizer 5A
- 200 Catalyst-C
- M-Bond 206 adhesive
- Sandpaper-320 grit
- CSM-2 degreaser
- Non-woven sponge
- 1 Pair dial calipers (65031464)

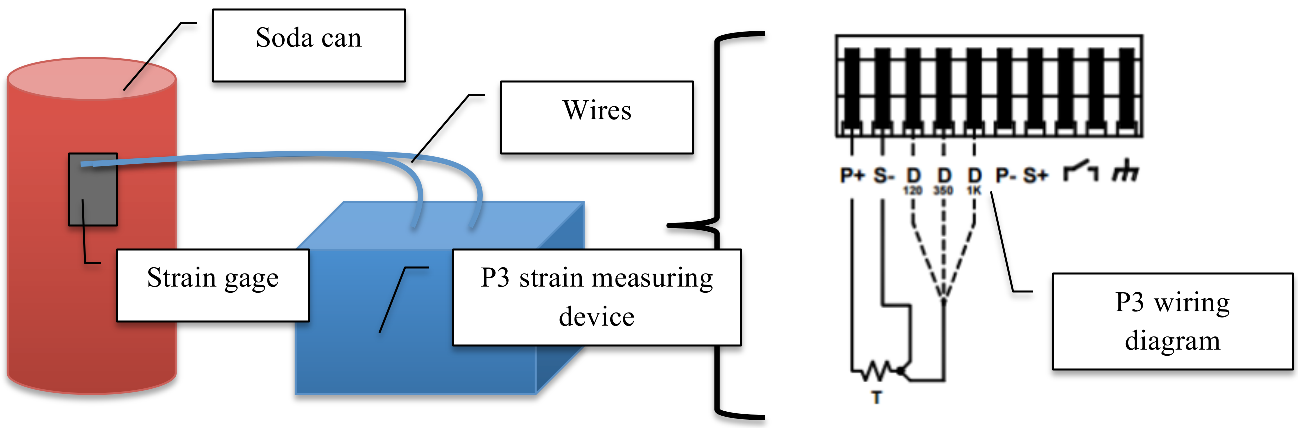

Figure 2.1. Experimental setup for measuring longitudinal stress per the lab handout. Wiring diagram to the right illustrates how to connect strain gage to P3 strain measurement system in quarter-bridge configuration (Credits: NC State Mechanical Engineering Laboratory I Handout).

Measuring strain on the soda can due to pressure relied upon the careful application of the strain gage. Using the procedure from the strain gage installation lab (Lab 4), a strain gage was attached to an unopened room temperature can of soda. Specifically, the procedure for surface preparation, gage bonding and applying solder to the solder tab of the strain gage for effective tinning was followed.

Then, using the procedure from the strain gage installation lab, lead wires were soldered to the strain gage on the soda can. Specifically, the ends of the stranded conductors were twisted tightly before tinning, making sure to have one end with three bare ends and the other with two -- black and white wires were twisted and tinned together. The bare wire was slowly drawn through the molten solder while continuously adding fresh solder to the interface of the wire and soldering tip. The tinned leadwire was trimmed. Leadwires were routed to the strain gage and firmly anchored to the test-part surface with drafting tape. Finally, the soldered connection was made.

Lastly, the strain gage was connected to P3 strain measurement system in a quarter-bridge configuration (see figure 2.1) per the lab handout. Then, the P3 was turned on and set to the correct channel. Next, the arrow controls were manipulated to set the wired channel for a quarter bridge. The arrow controls were then manipulated to set the correct gage factor. Finally, the strain gage was balanced using the "BAL" button to make strain read zero. At this point, the soda can was opened and the negative strain value recorded by the P3 was recorded. The radius and thickness of the soda can were measured using dial calipers, and the data was shared with the remainder of the section to create a larger pool of data.

3. Experimental Data

[Primary Contributor: David Delgado]

The modulus of elasticity given was:

- E = 10.4 x 10^6 psi

The Poisson's ratio was taken to be:

- v = 0.36

The individual strain measurements of different soda cans were collected and the correlating pressures were calculated (see Table 3.1):

Table 3.1. Strain and Pressure Measurements of Different Aluminum Cans

Brand	Strain (microstrain)	Strain Gage Placement	Diameter (in.)	Thickness (in.)	Pressure (psi)
Monster	-1065	Hoop	2.570	0.005	-52.558
Mountain Dew	-1394	Hoop	2.689	0.002	-26.300
Mountain Dew	-150	Longitudinal	2.498	0.012	-107.06
Monster	-167	Longitudinal	2.559	0.004	-38.783
Coke	-203	Longitudinal	2.19	0.004	-55.087
Coke	-252	Longitudinal	2.630	0.005	-71.179

The internal pressure utilizing the hoop strain was calculated as such:

$$P = \varepsilon_h \left(\frac{E}{1 - \frac{v}{2}} \right) \left(\frac{t}{r} \right) \; [psi] \tag{1}$$

On the other hand, the internal pressure utilizing the longitudinal strain was calculated using Equation 2. The experimental value of these pressures are summarized in Table 3.1 but one can find a more thorough analysis in the Sample Calculations section:

$$P = \varepsilon_l \left(\frac{E}{\frac{1}{2} - \nu} \right) \left(\frac{t}{r} \right) [psi] \tag{2}$$

4. Theory and Analysis

[Primary Contributor: David Delgado]

To evaluate the reliability of the body of the data, a statistical analysis was performed using Kline-McClintock's method based on Chauvenet's Criterion. The experimentally obtained values used to calculate the uncertainty for the pressure ω_p were thickness t, radius r, and the circumferential strain ε. The constants E (Young's Modulus for Aluminum Alloy 3004-H19) and ν (Poisson's ratio for the alloy) in the equation were found through an online source (Department of Defense) listed in the references. The uncertainty was calculated using the following equation(s):

$$\% \frac{\omega_p}{P} = \left[\left(\frac{\omega_{\varepsilon_h}}{\varepsilon_h} \right)^2 + \left(\frac{\omega_t}{t} \right)^2 + \left(-\frac{\omega_r}{r} \right)^2 \right]^{1/2} \times 100 \tag{1}$$

$$\% \frac{\omega_p}{P} = \left[\left(\frac{\omega_{\varepsilon_l}}{\varepsilon_l} \right)^2 + \left(\frac{\omega_t}{t} \right)^2 + \left(-\frac{\omega_r}{r} \right)^2 \right]^{1/2} \times 100 \tag{2}$$

An uncertainty of 44.954% was obtained from the pressure calculations based on axial strain (Equation 1). This is a relatively high percent of uncertainty, and leads to the conclusion that from these data values one cannot confidently determine the value of the pressure of a typical soda can. Furthermore, the uncertainty for pressure calculations based on longitudinal strain (Equation 2) was found to be 87.127%. Factors that may have contributed to this high percentage uncertainty will be discussed in the Discussion section.

Then, the same calculations were performed but omitting the pressure values based on Chauvenet's Criterion. Based on this criterion and calculations performed in the Sample Calculations section, the following pressure values were omitted from the analysis:

$$x_2, x_3 = -26.300\, psi, -107.06\, psi$$

The new uncertainty obtained for pressure calculations based on axial strain was 0%. The new uncertainty obtained for pressure calculations based on longitudinal strain was 28.654%. Evidently, the elimination of the outliers x_2 and x_3 improved the accuracy with which one could determine the pressure of a soda can.

5. Discussion

[Primary Contributor: David Delgado]

In the experimental measurements, the pressure value for Mountain Dew with longitudinal strain gage placement was removed from the calculations after it was determined that its d/σ (3.86) was greater than the z value (1.73) found for the number of data points. This was done following Chauvenet's Criterion, which states that values which have an experiment error function (d/σ) greater than z (dmax/σ) should be eliminated to inhibit incorrect interpretation of data. With this same approach, the pressure value (-26.300 με) for Mountain Dew with a horizontal strain gage placement was also removed from calculations. The percent differences for the internal pressures with respect to the average reading, before omitting the above pressure values, were calculated utilizing the following:

$$\% \, diff = \frac{\left|Value_{exp} - Value_{pub}\right|}{\frac{1}{2}(Value_{exp} + Value_{pub})} \times 100\% \quad (1)$$

The experimental percent difference values are show in Table 5.1, where $Value_{pub}$ is equal to the mean internal pressure of the data set (-58.495 psi) and $Value_{exp}$ is the individual internal pressure calculated for the different cans and strain gage placements.

Table 3.1. Strain and Pressure Measurements of Different Aluminum Cans

Brand	**Strain Gage Placement**	**Pressure (psi)**	**% Diff**
Monster	Hoop	-52.558	10.692%
Mountain Dew	Hoop	-26.300	75.936%
Mountain Dew	Longitudinal	-107.06	58.670%
Monster	Longitudinal	-38.783	40.527%
Coke	Longitudinal	-55.087	6.00%
Coke	Longitudinal	-71.179	19.563%

6. Conclusions

[Primary Contributor: David Delgado]

The high percentage in uncertainty suggests there may be an issue with the experimental method used for collecting data. The ratios between the standard deviations and means for the different terms (thickness, radius, strain) in the equation make it evident that there was higher error in thickness measurements. Specifically, the Mountain Dew thickness was almost four times greater than the average of the other cans. This is likely due to the greater difficulty lab groups had in reading aluminum thickness measurements from the micrometer tool.

The purpose of this lab was to provide knowledge regarding stress in a pressure vessel. Thus, the accuracy and lowering the uncertainty of the internal pressure calculations was a major concern. Further improvements to the experiment include standardizing the strain gage placement so that there is sufficient data for a statistical analysis on both placement methods. The current data does not provide a realistic uncertainty for internal pressure calculations based on axial strain. As shown by Table 3.1, both the hoop and longitudinal strain gage placements resulted in outlier pressure values. Thus, both methods for measuring stress seem equally valid as long as thickness and radius measurements were consistent from trial to trial. Still, axial strain computations appeared to result in the lowest deviation from the mean internal pressure.

However, the most likely case is that industry would implement the Chauvenet's Criterion as it takes significantly less to time calculate than the uncertainty calculations. Also this method allows one to obtain two sets of pressure values -- one before and one after the outlier(s) has been removed. Applying the Strain gage in the axial direction seems to be the best method as it provides the easiest way to calculate both pressure of the can and uncertainty values.

7. References

[Primary Contributor: David Delgado]

1) Department of Defense. (1998). Metallic Materials and Elements for Aerospace Vehicle Structures. MIL-HDBK-5J. Washington, D.C: AMSC.

2) Gere, J. & Goodno, B. (2013). Mechanics of materials. Stamford, Conn: Cengage Learning.

3) NC State Engineering. (2018). Lab Handout MAE 305 Experiment 6: Stress in a pressure vessel. Raleigh, N.C: NC State University.

4) Wheeler, A. & Ganji, A. (2010). Introduction to engineering experimentation. Upper Saddle River, N.J: Pearson Higher Education.

8. Sample Calculations

[Primary Contributor: David Delgado]

Several equations were used to properly conduct the lab.

Let the internal pressure utilizing the hoop strain be described by:

$$P = \varepsilon_h \left(\frac{E}{1 - \frac{\nu}{2}} \right) \left(\frac{t}{r} \right) \ [psi] \tag{1}$$

Where ε_h is the hoop strain, E is the modulus of elasticity of the aluminum specimen, ν is the Poisson's ratio, t is the mean thickness of the can and r is the mean radius of the can. Or in the case that longitudinal strain is used, the internal pressure was defined as:

$$P = \varepsilon_l \left(\frac{E}{\frac{1}{2} - \nu} \right) \left(\frac{t}{r} \right) [psi] \tag{2}$$

Where ε_l is the longitudinal strain, E is the modulus of elasticity of the aluminum specimen, ν is the Poisson's ratio, t is the mean thickness of the can and r is the mean radius of the can. The experimental values of the internal pressure of the cans were calculated as such:

$$P = \varepsilon_l\left(\frac{E}{\frac{1}{2}-\nu}\right)\left(\frac{t}{r}\right) = (-150\times10^{-6})\left(\frac{10.4\times10^{6}psi}{\frac{1}{2}-0.36}\right)\left(\frac{0.012in}{1.249in}\right) = -107.06\,psi \tag{3}$$

The statistical method used to justify the elimination of bad data points arising from poor experimentation was Chauvenet's Criterion. The method begins with the calculation of a mean value, deviation and standard deviation as follows:

$$x_m = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{4}$$

$$d_i = x_i - x_m \tag{5}$$

$$\sigma_{N-1} = \left\{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - x_m)^2\right\}^{1/2}, N < 20 \tag{6}$$

Where x_m is the mean value of the given set, d_i is the deviation of the singular values of the set and σ_{N-1} is the standard deviation of the set. The experimental value of these variables for the given set were defined as:

$$x_m = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{-52.558 - 26.3 - 107.06 - 38.783 - 55.087 - 71.179}{6} = -58.495\,psi \tag{7}$$

$$d_1 = x_1 - x_m = 5.937 \tag{8}$$

$$d_2 = x_2 - x_m = 32.195 \tag{9}$$

$$d_3 = x_3 - x_m = -48.565 \tag{10}$$

$$d_4 = x_4 - x_m = 19.712 \tag{11}$$

$$d_5 = x_5 - x_m = 3.408 \tag{12}$$

$$d_6 = x_6 - x_m = -12.684 \tag{13}$$

$$\sigma_{N-1} = \left\{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - x_m)^2\right\}^{1/2} = 12.580 \tag{14}$$

The last step in applying Chauvenet's Criterion was to eliminate all the data points having d_i/σ greater than d_{max}/σ. The experiment error function was calculated for six (6) data points and the experimental value was calculated as:

$$z \equiv \frac{d_{max}}{\sigma} = 1.73 \tag{15}$$

This value was used to eliminate the following data points from the set:

$$x_2, x_3 = -26.300\, psi, -107.06\, psi \tag{16}$$

In finding the uncertainty of the data, the Kline and McClintock Uncertainty Analysis method was used:

$$\% \frac{\omega_p}{P} = \left[\left(\frac{\omega_{\varepsilon_h}}{\varepsilon_h}\right)^2 + \left(\frac{\omega_E}{E}\right)^2 + \left(\frac{\omega_v}{\frac{1}{2(1-\frac{v}{2})}}\right)^2 + \left(\frac{\omega_t}{t}\right)^2 + \left(-\frac{\omega_r}{r}\right)^2\right]^{1/2} \times 100 \tag{17}$$

Where ω_i is the deviation of the strain, modulus of elasticity, thickness, radius, and Poisson's ratio measurement. Thus, the equation above can be simplified to the following form:

$$\% \frac{\omega_p}{P} = \left[\left(\frac{\omega_{\varepsilon_h}}{\varepsilon_h}\right)^2 + \left(\frac{\omega_t}{t}\right)^2 + \left(-\frac{\omega_r}{r}\right)^2\right]^{1/2} \times 100 \tag{18}$$

Where the experimental value of pressure uncertainty for the axial strain was calculated as:

$$\% \frac{\omega_p}{P} = \left[\left(\frac{164.5}{1229.5}\right)^2 + \left(\frac{0.0015}{0.0035}\right)^2 + \left(-\frac{0.02975}{1.31475}\right)^2\right]^{1/2} \times 100 = 44.954\% \tag{19}$$

Omitting the values based on Chauvenet's Criterion:

$$\% \frac{\omega_p}{P} = \left[\left(\frac{0}{1065}\right)^2 + \left(\frac{0}{0.005}\right)^2 + \left(-\frac{0}{1.285}\right)^2\right]^{1/2} \times 100 = 0\% \tag{20}$$

Similarly, the uncertainty analysis for longitudinal strain was conducted using the following equation:

$$\% \frac{\omega_p}{P} = \left[\left(\frac{\omega_{\varepsilon_l}}{\varepsilon_l}\right)^2 + \left(\frac{\omega_t}{t}\right)^2 + \left(-\frac{\omega_r}{r}\right)^2\right]^{1/2} \times 100 \tag{21}$$

Where the experimental of pressure uncertainty for the longitudinal strain was calculated as:

$$\% \frac{\omega_p}{P} = \left[\left(\frac{59}{193}\right)^2 + \left(\frac{0.00505}{0.00625}\right)^2 + \left(-\frac{0.139625}{1.234625}\right)^2\right]^{1/2} \times 100 = 87.127\% \tag{22}$$

Again, omitting the values based on Chauvenet's Criterion:

$$\% \frac{\omega_p}{P} = \left[\left(\frac{44.667}{207.33}\right)^2 + \left(\frac{0.00066667}{0.0043333}\right)^2 + \left(-\frac{0.13483}{1.2298}\right)^2\right]^{1/2} \times 100 = 28.654\% \tag{23}$$