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Short Form Laboratory Report

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Title: Lab 7, Natural and Forced Convection of a Heat Sink
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1. Abstract

The objective of this experiment is to investigate the underlying principles and mechanics involved in the natural and forced heat convection of a standard heat sink. This was conducted through a Peltier chip on a finned assembly as the heat sink, with forced conduction done through the addition of a fan. The ability to control temperature through heat convection in temperature sensitive devices is highly significant in real world applications. These can include anything from motherboard chips to heavy machinery under stress. For this particular experiment, we tested the thermal performance of the finned assembly for forced and free convection. This was allowed to proceed until steady state was achieved, and thermocouples placed strategically throughout the fins measured the temperatures at different locations throughout the duration. This recorded data was then analyzed through MATLAB to determine the convection coefficient for both the forced and free heat convection and allow them to be compared in their efficiency.

Completion of the experiment and evaluation of the data collected allowed a comparison to be drawn between the heat transfer rates and efficiencies of the free and forced convection setups. First, the convective coefficient, *h*, was determined at each thermocouple location and with respect to time. The efficiency of each method (free or forced flow) was determined from these values. After the data was finalized and plotted, some trends between the variables were apparent. In short, the addition of a fan, and thus an increased convective heat transfer coefficient, greatly improves the rate at which heat is dissipated by the aluminum heat sink. Furthermore, this increased dissipation of heat resulting from the fan makes for lower fin efficiency values for the forced convection experiment due to the fact that this improved thermal performance is not a surface efficiency phenomenon.

2. Equations

The following equations were used to perform the analysis below:

First, Newton's Law of Cooling was used to determine the heat transferred via free and forced convection.

$$q = hA_s(T_s - T_\infty) \qquad \textbf{(1)}$$

In this equation, q is the heat transferred via convection, A_s is the area of the surface in contact with the fluid, T_s is the temperature of the surface and T_∞ is the temperature of the fluid (i.e., air at room temperature). The next equation utilized was a representation of the temperature distribution along a fin on the heat sink:

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL} \qquad \textbf{(2)}$$

Where

$$\theta \equiv T - T_\infty$$

$$\theta_b \equiv T_b - T_\infty$$

$$m = \sqrt{hP/kA_c}$$

In equation 2, L is the length of the fin from the base to the tip, x is the distance from the base to the location of the temperature measurement (i.e., the thermocouple), h is the convection coefficient, k is the conduction coefficient of the fin material (in the case of this experiment, k = 167 W/mK), T is the temperature measured, T_b is the temperature of the base, P is the perimeter of the fin base cross-section, and A_c is the base cross-sectional area of the fin.

Once equation 2 was used to solve for the convection coefficient, the experiment called for the calculation of the fin heat transfer rate, q_f:

$$q_f = M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} \qquad \textbf{(3)}$$

Where

$$M = \sqrt{hPkA_c}\,\theta_b$$

Next, the performance impact of the individual fins and the heat sink as a whole was determined via the equations 4 and 5. The fin effectiveness, ε_f, was found using the following:

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} \qquad \textbf{(4)}$$

In this equation, $A_{c,b}$ is the fin cross-sectional area at the base. In the case of the heat sink utilized in this experiment, $A_{c,b}$ and A_c are equivalent due to the assumed uniform cross section of the rectangular fins. Then, the fin efficiency, η_f, was calculated using the following:

$$\eta_f = \frac{q_f}{hA_f\theta_b} \qquad \textbf{(5)}$$

Where A_f is the area exposed to the fluid, or rather the surface area of the fin. In short, equations 3 through 5 were used to study the effect of a single fin. The effect and efficiency of the entire heat sink was studied using similar equations. For instance, the total heat transfer rate for the whole heat sink, q_t, was calculated using the following:

$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b \qquad \textbf{(6a)}$$

$$q_t = h[N\eta_f A_f + (A_t - NA_f)]\theta_b \qquad \textbf{(6b)}$$

$$q_t = hA_t[1 - \frac{NA_f}{A_t}(1 - \eta_f)]\theta_b \qquad \textbf{(6c)}$$

In equation 6c, N is the number of fins (in this case 14) and A_t is the total surface area. Lastly, the overall surface efficiency, η_o, was calculated using the following relationship:

$$\eta_o = \frac{q_t}{hA_t\theta_b} \qquad \textbf{(7)}$$

In summary, the above equations were used to analyze the effect and the efficiency of the given heat sink. From the resulting data it was determined whether or not the heat sink provides enough cooling for the application.

3. Experimental Data & Results

Table 1: Physical Measurements of Heat Sink

1 - Fin Thickness (m)				**2 - Fin Gap Thickness (m)**			
1	0.0020828	8	0.0010668	1	0.0032004	8	0.0041148
2	0.0020066	9	0.0011176	2	0.0041148	9	0.0041148
3	0.0011176	10	0.001143	3	0.0123698	10	0.0123698
4	0.0010668	11	0.0011176	4	0.0123698	11	0.0123698
5	0.0010668	12	0.0010922	5	0.0041148	12	0.0041148

6	0.001143	13	0.0020066	6	0.0041148	13	0.0032004
7	0.0011176	14	0.001905	7	0.0041148		
Average		0.0013607		Average		0.0065141	
3 - Height, Width and Depth (m)				**4 - Thermocouple Positions (m)**			
Height		0.040310		From Fin Base		From Side	
Width		0.089789		T1	At the base	T1	At the base
Depth		0.090830		T2	0.0056779	T2	0.067882
				T3	0.013913	T3	0.045491
				T4	0.023191	T4	0.023698

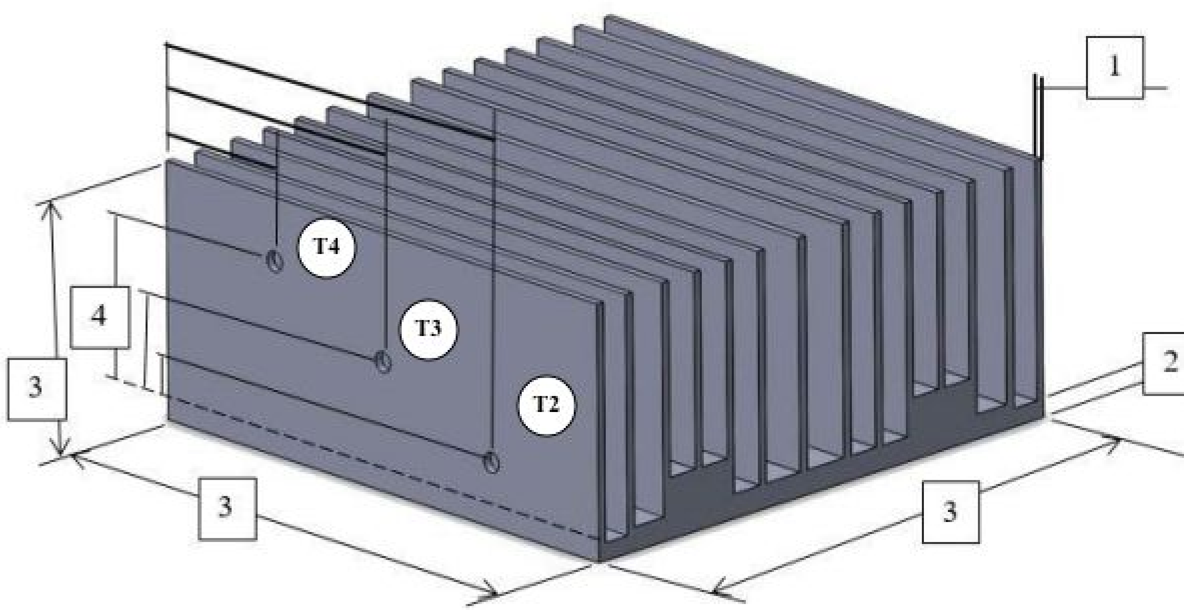

Figure 1: Heat Sink Measurement Locations

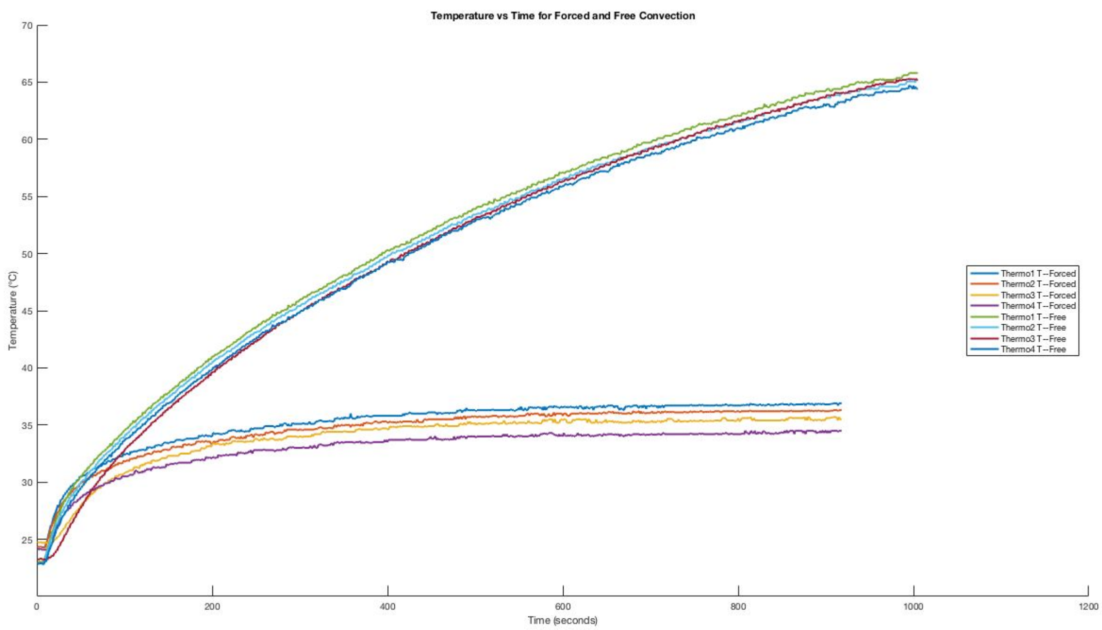

Figure 2: Temperature vs Time for Forced and Free Convection Measured at Thermocouples

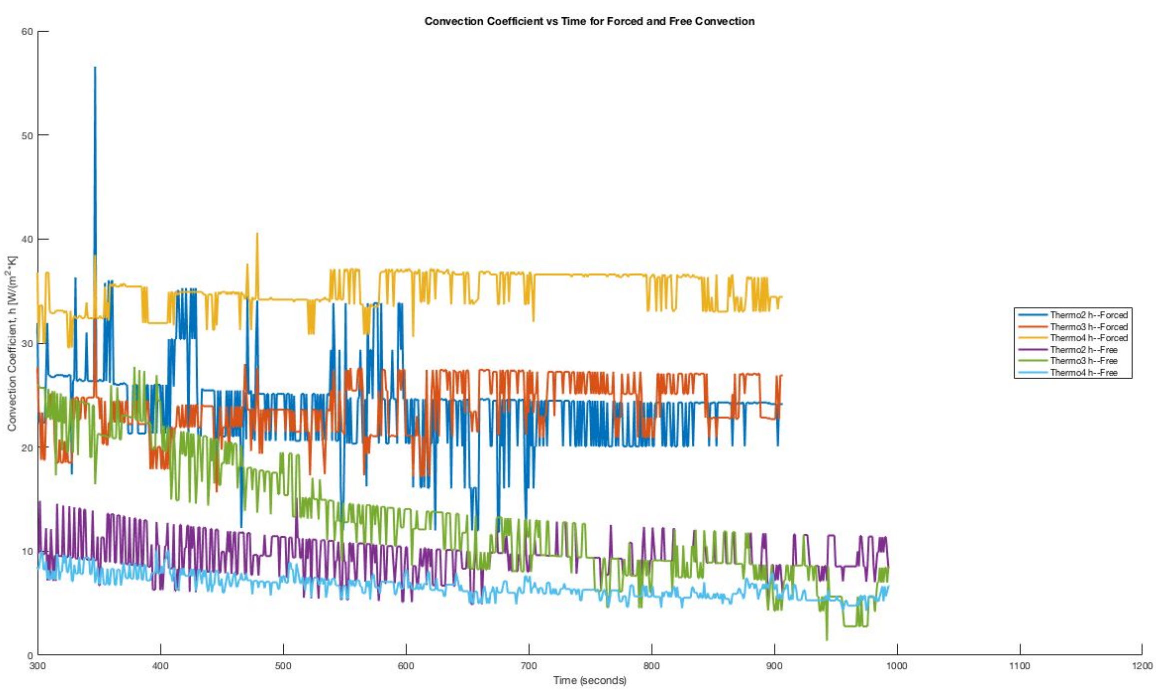

Figure 3: Convection Coefficient vs Time for Forced and Free Convection

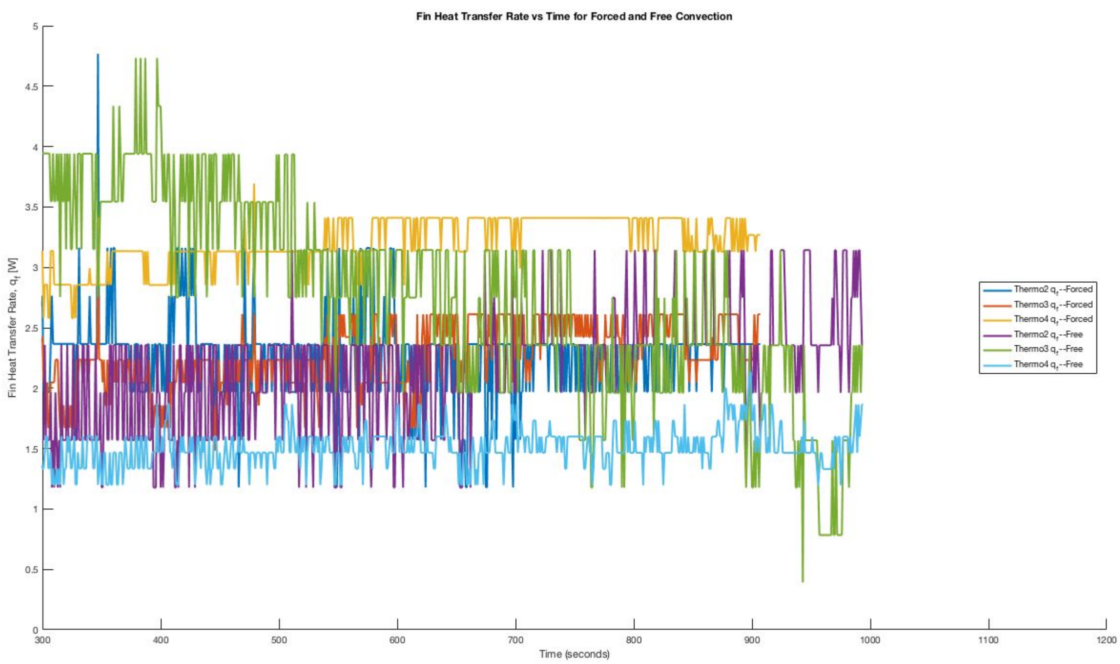

Figure 4: Fin Heat Transfer Rate vs Time for Forced and Free Convection

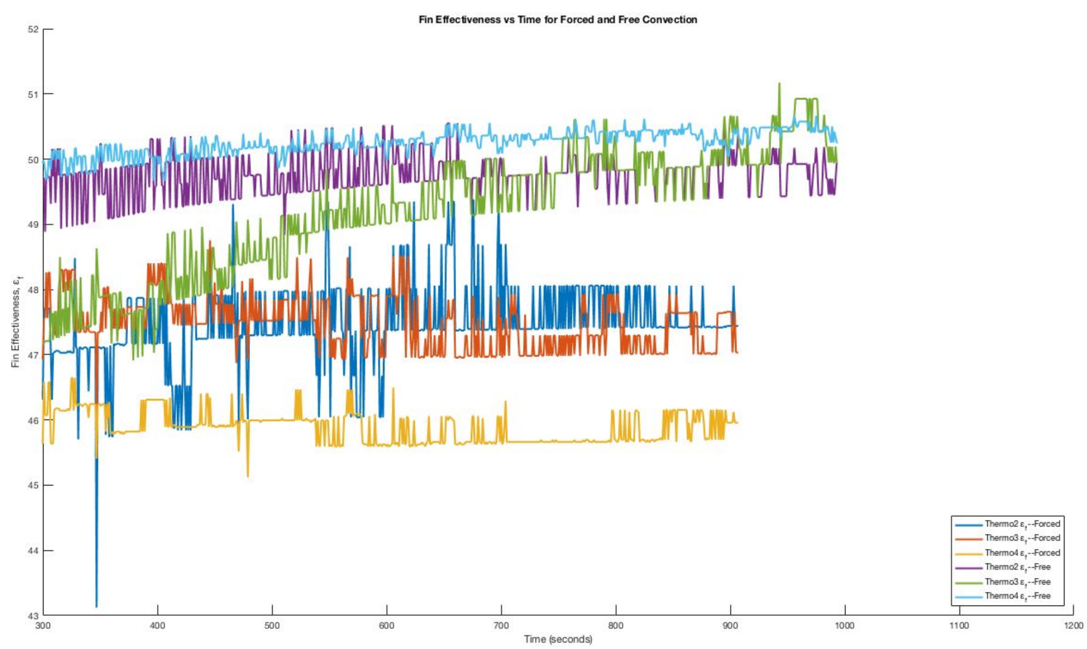

Figure 5: Fin Effectiveness vs Time for Forced and Free Convection

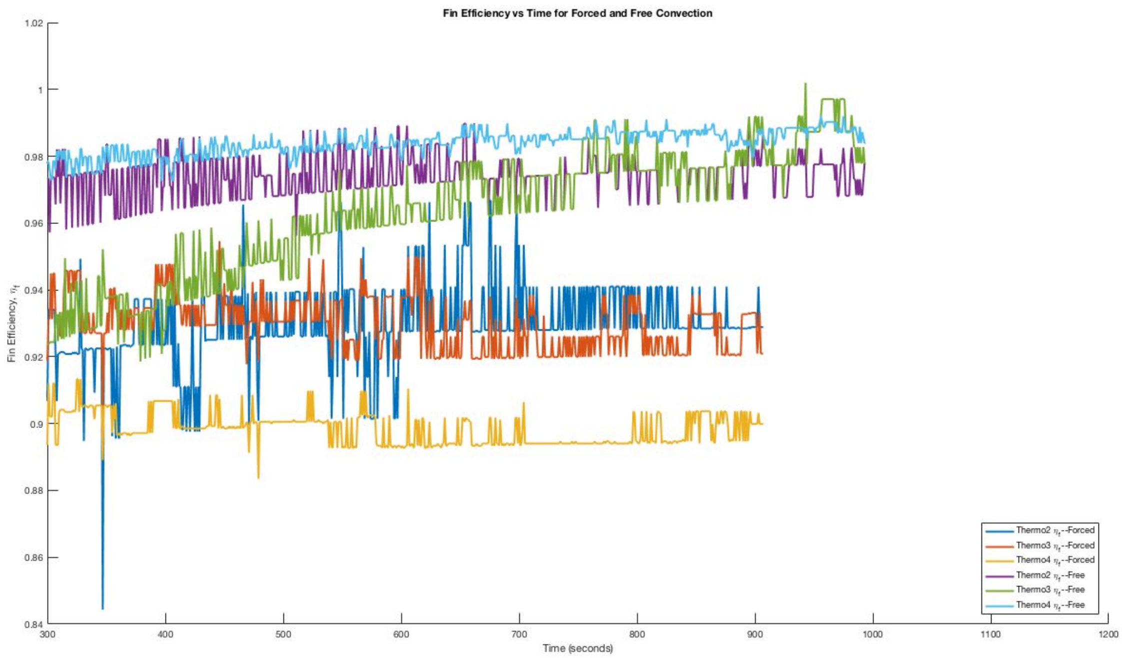

Figure 6: Fin Efficiency vs Time for Forced and Free Convection

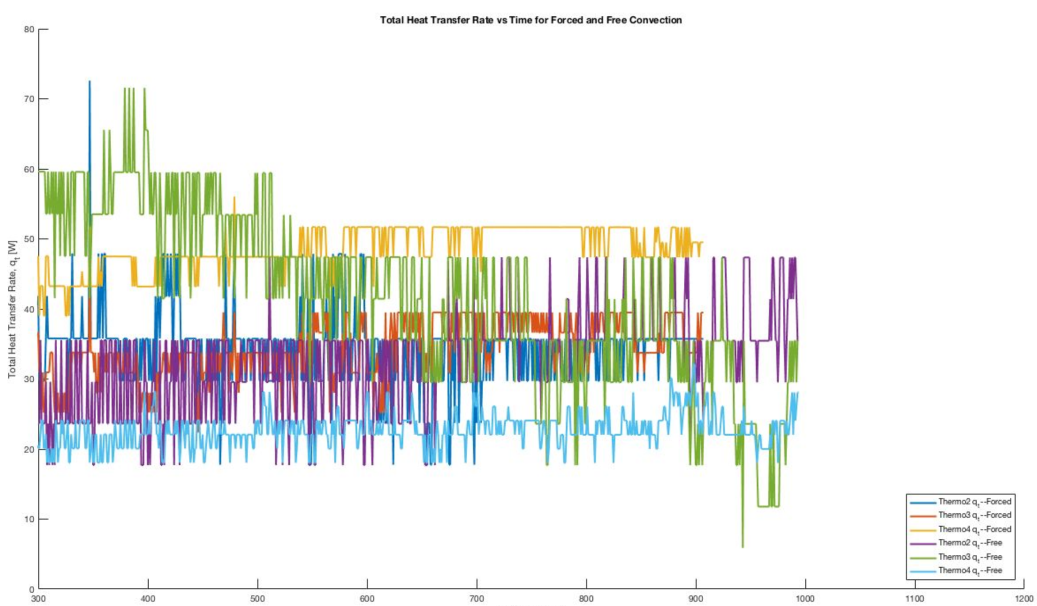

Figure 7: Total Heat Transfer Rate vs Time for Forced and Free Convection

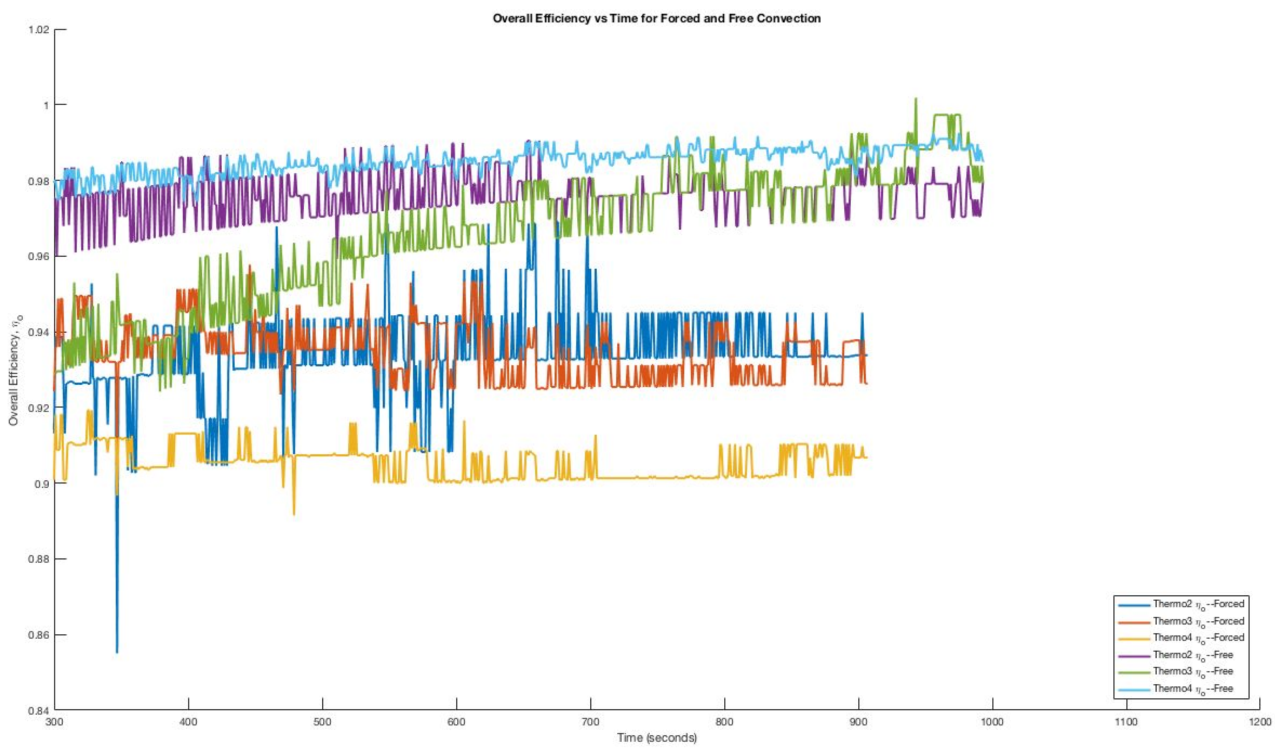

Figure 8: Overall Surface Efficiency vs Time for Forced and Free Convection

4. Discussion

Equation 2 shows that as x (distance traveled along the fin) increases, the size of the temperature gradient decreases. This relationship between x and the temperature gradient is the result of the relationship between x and heat transfer $q_x(x)$. As x increases, heat transfer decreases because of the convection losses from the fins' surfaces. These convection losses are present in both free convection and forced convection models. In free convection, the movement of the fluid is due entirely to density gradients within the fluid. In other words, hot air rises over cold air as seen in figure 9. There is no external device or phenomenon which causes fluid motion. In forced convection, the fluid is forced to flow by an external factor (i.e., a fan blowing air over the heat sink) as seen in figure 10. Typically heat transfer under forced convection conditions is higher than natural convection for the same fluid. Still, the underlying assumption in the analysis of forced convection is that the effects of free convection are negligible for this case. This, of course, is not true in practice because free convection is likely when there is an unstable temperature gradient. In this way, situations may arise for which free and forced convection effects are comparable, in which case it is inappropriate to neglect either process. For instance, in the case of assisting flows, buoyancy acts to enhance the rate of heat transfer associated with pure forced convection. Assisting flow refers to the case in which buoyancy-induced and forced motions have the same direction. Therefore, the flows studied in this experiment are quite intricate, which complicates heat transfer predictions.

Furthermore, although buoyancy effects can significantly enhance heat transfer for laminar forced convection flows, enhancement is typically negligible if the forced flow is turbulent. For this reason, it is impossible to characterize the flow of the forced convection model in this experiment without determining whether the flow induced by forced convection is laminar or turbulent. Nevertheless, this lab will attempt to describe the trends in the data that show up on each plot.

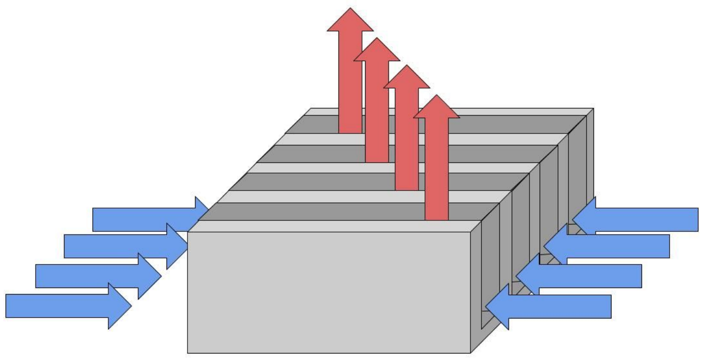

Figure 9: Free Convective Heat Transfer Model

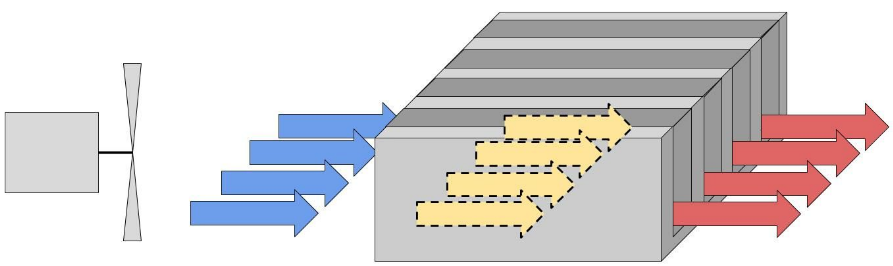

Figure 10: Forced Convective Heat Transfer Model

The data collected during the lab shows that the addition of a fan, and thus an increased convective heat transfer coefficient greatly improves the rate at which heat is dissipated by the aluminum heat sink. The experimental runs that utilize the fan have a much smaller maximum temperature than the runs that do not utilize the fan, shown in Figure 2. All of the runs, however, share a similar logarithmic trend in their temperature over time data, decreasingly increasing until reaching steady state and platoaing to a maximum temperature as time continues.

It is interesting to note that in figure 2 at points below 150 seconds for forced convection and 300 seconds for free convection, there are instances where the temperature readings at thermocouple 3 are less than those at thermocouple 4. This trend is not expected for transient conduction along the heat sink, since points further from fin base should read lower temperatures. Thus, a limit was used on the x-axis in order to capture trends only above 300 seconds for the figures 3-8. Through analysis of figure 2 and logical deduction, it was also determined that our original thermocouple location had been misrecorded; hence it was decided that thermocouple 1 and 3 had to be switched. This is due to the fact that the labeled thermocouple 3 read the highest temperatures and therefore should have been labeled thermocouple 1. The thermocouple closest to the base will always have the highest temperatures as it is closest to the heat source. This affected the entire MATLab code until the error was corrected. It is believed that the input device for the myRIO simply may have had one of the inputs switched, causing the misrecording. Figure 2 in this report is the corrected plot of temperature vs time for forced and free convection. Figure 3-8 were constructed based on the modifications described above.

From figure 3, it can be observed that the convection coefficient is higher for the forced convection experiment as expected. In theory, a higher h value should translate into the higher heat transfer rates. However, Newton's Law of Cooling states that the heat rate also depends on the temperature difference between the base of the heat sink and the surrounding air. Since the base temperatures for the free convection case reach much higher values (see figure 2), a higher heat transfer rate can be expected for the free convection case until the point that the convection coefficient associated with free convection decreases. At this point, the conclusion that heat transfer under forced convection conditions is higher than free convection for the same fluid is satisfied. This is due to the fact that free convection flow velocities are generally much smaller than those associated with forced convection, and therefore the corresponding convection transfer rates are also smaller.

Figure 4 helps to confirm an expected trend, that higher convective heat transfer rates are expected for the forced convection experiment due to higher flow velocities. Similarly to Figure 4, Figure 7 shows total heat transfer rate and shows that the addition of the fan seems to make the heat transfer rate more consistent with less deviation. The completely free convection appears to oscillate more as the experiment was conducted. However, this conclusion cannot be made without further investigation into the actual air flow velocity. Nevertheless, higher convective heat transfer occurs for the forced convection experiment, as expected, due to higher flow velocities. Again, the exact effect cannot be calculated theoretically without knowing whether the flow is laminar or turbulent.

Since the addition of a fin assembly does not automatically assure an effective heat convection, an analysis of the fins must be done through fin effectiveness. A fin effectiveness of 2 or greater is desired for the application of fins to be practical. Interestingly, the fin effectiveness is generally higher for free convection. This might be attributed to the lower convection coefficients associated with free convection compared to forced convection -- i.e., $\varepsilon_f \sim 1/h$. Hence, the benefits of fins in terms of increasing the heat transfer rate are more conspicuous for the free convection experiment. This fin effectiveness is plotted in figure 5.The fins in this experiment were to be determined to be sufficiently high enough for the desired application.

Fin efficiency is graphed in Figure 6. It should be noted that in general the fin efficiency for each trial is considerably high (i.e., in the 90% range). The results also show us that η_f approaches its maximum value of 1 as the distance from the base is at a minimum. In other words, for both free and forced convection, thermocouple 2 sees higher fin efficiency values on average. Furthermore, the fin efficiency for free convection tends to be higher than that of forced convection due to the fact that free convection is associated with lower convection coefficient values -- i.e., $\eta_f \sim 1/h$. Hence, the augmented fin thermal performance is more conspicuous for the free convection experiment.

Similarly to Figure 6, Figure 8 graphs overall surface efficiency, and shows that overall surface efficiency is lower for the forced convection than it is for free convection. Thus, the trends seen in this figure are very similar to those of individual fin effincies as expected (see figure 6). This is due to the fact that forced convection already increases the heat transfer rate considerably, and therefore adding extended surfaces does not improve the fin thermal performance to the degree that it is improved for free convection. While the forced convection is better at transferring heat by convection and will keep the device cooler, this performance does not come from the surface efficiency.

5. Conclusion

This lab elucidates the enormous advantage provided by extended surfaces in terms of increasing the heat transfer between a surface and a fluid. Newton’s Law of Cooling (Equation 1) shows that there are only three ways to increase the convective heat transfer. One option is to increase the temperature difference (i.e., T_s - T_∞) between the surface and the fluid by means of cooling the fluid. In the case of the heat sink assembly, a cooling system to achieve this is not a practical solution due to limited space. A second option is to increase the convective heat transfer coefficient (i.e., h) by increasing the fluid velocity via a larger fan. Once again, this solution is not practical due to the increased power that would be required to operate a larger fan and limited space in the processor. The remaining option is to increase the surface area (i.e., A_s), which is the purpose of the finned heat sink utilized in this experiment. The comparison of forced and free convection over a finned surface leads us to the conclusion that the addition of a fan, and thus an increased convective heat transfer coefficient, greatly improves the rate at which heat is dissipated by the aluminum heat sink. Furthermore, this increased dissipation of heat resulting from the fan makes for lower fin efficiency values for the forced convection experiment due to the fact that this improved thermal performance is not a surface efficiency phenomenon.

6. Appendix

```
% Code for convection lab
clear
clc
close all
k = 167; % conduction coefficient of fin from lab handout
N = 14; % Number of fins
P = 0.18230; % perimeter of the fin base cross-section , P = 2*fin thickness + 2*width of heat sink
Ac = .00012218; % base cross-sectional area of the fin , Ac = avg fin thickness*width of heat sink
Acb = .00012218; % fin cross-sectional area at the base , Acb = Ac
L = .033796; % length of fin from base to tip , L = height - avg fin gap thickness
Af = 0.0062401; % area exposed to fluid OR total surface area of the fin , Af = 2*width*(L+t/2)
Ab = 0.0064451; % area of exposed base , Ab = width*depth - N*Ac
At = 0.093806; % total surface area At = N*Af+Ab
Tinf = 20; % fluid temp
% Import Experimental Results
Tx = csvread('Forced Convection.csv');
T3_fo = Tx(:,2);
T2_fo = Tx(:,3);
T1_fo = Tx(:,4);
T4_fo = Tx(:,5);
Tx2 = csvread('Free Convection.csv');
T3_fr = Tx2(:,2);
T2_fr = Tx2(:,3);
T1_fr = Tx2(:,4);
T4_fr = Tx2(:,5);

% Forced Convection
tTL = length(T1_fo)*3;
% Preallocating for the forced convection for-loop
hT2 = zeros(1,length(T1_fo)); % convection coefficient at thermocouple 2
hT3 = zeros(1,length(T1_fo)); % convection coefficient at thermocouple 3
hT4 = zeros(1,length(T1_fo)); % convection coefficient at thermocouple 4
qFT2 = zeros(1,length(T1_fo));
qFT3 = zeros(1,length(T1_fo)); % qf - heat transfer rate of single fin
qFT4 = zeros(1,length(T1_fo));
fINeFFECTt2 = zeros(1,length(T1_fo));
fINeFFECTt3 = zeros(1,length(T1_fo)); % fin effectiveness
fINeFFECTt4 = zeros(1,length(T1_fo));
fINeFFICt2 = zeros(1,length(T1_fo));
fINeFFICt3 = zeros(1,length(T1_fo)); % fin efficiency
fINeFFICt4 = zeros(1,length(T1_fo));
qTt2 = zeros(1,length(T1_fo));
```

```
qTt3 = zeros(1,length(T1_fo)); % qt - total heat transfer rate
qTt4 = zeros(1,length(T1_fo));
oVReFFICt2 = zeros(1,length(T1_fo));
oVReFFICt3 = zeros(1,length(T1_fo)); % overall surfact efficiency
oVReFFICt4 = zeros(1,length(T1_fo));
% Initialize for loop to determine the values above
for i=1:tTL
  % Evaluating for Thermocouple 2
  if (i >= 0) && (i <= (tTL/3))
    x = 0.0056779;
    Tb = T1_fo(i); % temperature of base is the temperature at thermocouple 1
    T = T2_fo(i); % temperature recorded by thermocouple 2
    tHETA = (T-Tinf);
    tHETAb = (Tb-Tinf);
    % Evaluating h
    syms h1
    m = sqrt(h1.*P./(k.*Ac));
    S = (tHETA./tHETAb)==((cosh(m*(L-x)))+(h1./(m*k))*(sinh(m*(L-x))))...
      /((cosh(m*L))+(h1./(m*k))*(sinh(m*L)));
    hT2(i) = vpasolve(S,h1,20);
    % solve for other parameters using an individual convection
    % coefficient (h)
    h = hT2(i);
    % Finding m and M
    m = sqrt(h.*P./(k.*Ac));
    M = sqrt(h.*P.*k.*Ac).*(tHETAb);
    % Find fin heat transfer rate (qF) and tabulate values at
    % thermocouple for plotting
    qFT2(i) = (M.*(sinh(m.*L)+(h./(m.*k))*cosh(m.*L)))/(cosh(m.*L)+(h./(m.*k))*sinh(m.*L));
    qF = qFT2(i);
    % Evaluate Fin Effectiveness (epsilon)
    fINeFFECTt2(i) = qF./(h.*Ac.*tHETAb);
    % Evaluate Fin Efficiency
    fINeFFICt2(i) = qF./(h.*Af.*tHETAb);
    fINeFFIC = fINeFFICt2(i);
    % Evaluate total heat transfer rate (qT)
    qTt2(i) = h.*At.*(1-(((N.*Af)/At)*(1-fINeFFIC)))*tHETAb;
    qT = qTt2(i);
    % Evaluate Overall Efficiency
    oVReFFICt2(i) = qT./(h.*At.*(tHETAb));
  % Evaluating for Thermocouple 3
  elseif (i > (tTL/3)) && (i <= ((2*tTL)/3))
    x = 0.013913;
```

```
        Tb = T1_fo(i-(tTL/3));
        T = T3_fo(i-(tTL/3));
        tHETA = (T-Tinf);
        tHETAb = (Tb-Tinf);
        % Evaluating h
        syms h1
        m = sqrt(h1.*P./(k.*Ac));
        S = (tHETA./tHETAb)==((cosh(m*(L-x)))+(h1./(m*k))*(sinh(m*(L-x))))...
            /((cosh(m*L))+(h1./(m*k))*(sinh(m*L)));
        hT3(i-(tTL/3)) = vpasolve(S,h1,20);
        % solve for other parameters using an individual convection
        % coefficient (h)
        h = hT3(i-(tTL/3));
        % Finding m and M
        m = sqrt(h.*P./(k.*Ac));
        M = sqrt(h.*P.*k.*Ac).*(tHETAb);
        % Find fin heat transfer rate (qF) and tabulate values at
        % thermocouple for plotting
                                                        qFT3(i-(tTL/3))         =
(M.*(sinh(m.*L)+(h./(m.*k))*cosh(m.*L)))/(cosh(m.*L)+(h./(m.*k))*sinh(m.*L));
        qF = qFT3(i-(tTL/3));
        % Evaluate Fin Effectiveness (epsilon)
        fINeFFECTt3(i-(tTL/3)) = qF./(h.*Ac.*tHETAb);
        % Evaluate Fin Efficiency
        fINeFFICt3(i-(tTL/3)) = qF./(h.*Af.*tHETAb);
        fINeFFIC = fINeFFICt3(i-(tTL/3));
        % Evaluate total heat transfer rate (qT)
        qTt3(i-(tTL/3)) = h.*At.*(1-(((N.*Af)/At)*(1-fINeFFIC)))*tHETAb;
        qT = qTt3(i-(tTL/3));
        % Evaluate Overall Efficiency
        oVReFFICt3(i-(tTL/3)) = qT./(h.*At.*(tHETAb));
    % Evaluating for Thermocouple 4
    elseif (i >= ((2*tTL)/3)) && (i <= tTL)
        x = 0.023191;
        Tb = T1_fo(i-((2*tTL)/3));
        T = T4_fo(i-((2*tTL)/3));
        tHETA = (T-Tinf);
        tHETAb = (Tb-Tinf);
        % Evaluating h
        syms h1
        m = sqrt(h1.*P./(k.*Ac));
        S = (tHETA./tHETAb)==((cosh(m*(L-x)))+(h1./(m*k))*(sinh(m*(L-x))))...
            /((cosh(m*L))+(h1./(m*k))*(sinh(m*L)));
```

```
        hT4(i-((2*tTL)/3)) = vpasolve(S,h1,20);
        % solve for other parameters using an individual convection
        % coefficient (h)
        h = hT4(i-((2*tTL)/3));
        % Finding m and M
        m = sqrt(h.*P./(k.*Ac));
        M = sqrt(h.*P.*k.*Ac).*(tHETAb);
        % Find fin heat transfer rate (qF) and tabulate values at
        % thermocouple for plotting
        qFT4(i-((2*tTL)/3)) =
(M.*(sinh(m.*L)+(h./(m.*k))*cosh(m.*L)))/(cosh(m.*L)+(h./(m.*k))*sinh(m.*L));
        qF = qFT4(i-((2*tTL)/3));
        % Evaluate Fin Effectiveness (epsilon)
        fINeFFECTt4(i-((2*tTL)/3)) = qF./(h.*Ac.*tHETAb);
        % Evaluate Fin Efficiency
        fINeFFICt4(i-((2*tTL)/3)) = qF./(h.*Af.*tHETAb);
        fINeFFIC = fINeFFICt4(i-((2*tTL)/3));
        % Evaluate total heat transfer rate (qT)
        qTt4(i-((2*tTL)/3)) = h.*At.*(1-(((N.*Af)/At)*(1-fINeFFIC)))*tHETAb;
        qT = qTt4(i-((2*tTL)/3));
        % Evaluate Overall Efficiency
        oVReFFICt4(i-((2*tTL)/3)) = qT./(h.*At.*(tHETAb));
    end
end

% Free Convection
tTL1 = length(T1_fr)*3;
% Preallocating for the free convection for-loop
hT21 = zeros(1,length(T1_fr));
hT31 = zeros(1,length(T1_fr));
hT41 = zeros(1,length(T1_fr));
qFT21 = zeros(1,length(T1_fr));
qFT31 = zeros(1,length(T1_fr));
qFT41 = zeros(1,length(T1_fr));
fINeFFECTt21 = zeros(1,length(T1_fr));
fINeFFECTt31 = zeros(1,length(T1_fr));
fINeFFECTt41 = zeros(1,length(T1_fr));
fINeFFICt21 = zeros(1,length(T1_fr));
fINeFFICt31 = zeros(1,length(T1_fr));
fINeFFICt41 = zeros(1,length(T1_fr));
qTt21 = zeros(1,length(T1_fr));
qTt31 = zeros(1,length(T1_fr));
qTt41 = zeros(1,length(T1_fr));
```

```
oVReFFICt21 = zeros(1,length(T1_fr));
oVReFFICt31 = zeros(1,length(T1_fr));
oVReFFICt41 = zeros(1,length(T1_fr));
for i=1:tTL1
  % Evaluating for Thermocouple 2
  if (i >= 0) && (i <= (tTL1/3))
    x = 0.0056779;
    Tb = T1_fr(i);
    T = T2_fr(i);
    tHETA = (T-Tinf);
    tHETAb = (Tb-Tinf);
    % Evaluating h
    syms h1
    m = sqrt(h1.*P./(k.*Ac));
    S = (tHETA./tHETAb)==((cosh(m*(L-x)))+(h1./(m*k))*(sinh(m*(L-x))))...
      /((cosh(m*L))+(h1./(m*k))*(sinh(m*L)));
    hT21(i) = vpasolve(S,h1,20);
    % solve for other parameters using an individual convection
    % coefficient (h)
    h = hT21(i);
    % Find m and M
    m = sqrt(h.*P./(k.*Ac));
    M = sqrt(h.*P.*k.*Ac).*(tHETAb);
    % Find fin heat transfer rate (qF) and tabulate values at
    % thermocouple for plotting
    qFT21(i) = (M.*(sinh(m.*L)+(h./(m.*k))*cosh(m.*L)))/(cosh(m.*L)+(h./(m.*k))*sinh(m.*L));
    qF = qFT21(i);
    % Evaluate Fin Effectiveness (epsilon)
    fINeFFECTt21(i) = qF./(h.*Ac.*tHETAb);
    % Evaluate Fin Efficiency
    fINeFFICt21(i) = qF./(h.*Af.*tHETAb);
    fINeFFIC = fINeFFICt21(i);
    % Evaluate total heat transfer rate (qT)
    qTt21(i) = h.*At.*(1-(((N.*Af)/At)*(1-fINeFFIC)))*tHETAb;
    qT = qTt21(i);
    % Evaluating Overall Efficiency
    oVReFFICt21(i) = qT./(h.*At.*(tHETAb));
  % Evaluating for Thermocouple 3
  elseif (i > (tTL1/3)) && (i <= ((2*tTL1)/3))
    Tb = T1_fr(i-(tTL1/3));
    T = T3_fr(i-(tTL1/3));
    tHETA = (T-Tinf);
    tHETAb = (Tb-Tinf);
```

```
        % Evaluating h
T31(i-(tTL1/3));
        % Evaluate Fin Effectiveness (epsilon)
        fINeFFECTt31(i-(tTL1/3)) = qF./(h.*Ac.*tHETAb);
        % Evaluate Fin Efficiency
        fINeFFICt31(i-(tTL1/3)) = qF./(h.*Af.*tHETAb);
        fINeFFIC = fINeFFICt31(i-(tTL1/3));
        % Evaluate total heat transfer rate (qT)
        qTt31(i-(tTL1/3)) = h.*At.*(1-(((N.*Af)/At)*(1-fINeFFIC)))*tHETAb;
        qT = qTt31(i-(tTL1/3));
        % Evaluating Overall Efficiency
        oVReFFICt31(i-(tTL1/3)) = qT./(h.*At.*(tHETAb));
    % Evaluating for Thermocouple 4
    elseif (i >= ((2*tTL1)/3)) && (i <= tTL1)
        x = 0.023191;
        Tb = T1_fr(i-((2*tTL1)/3));
        T = T4_fr(i-((2*tTL1)/3));
        tHETA = (T-Tinf);
        tHETAb = (Tb-Tinf);
        % Evaluating h
        syms h1
        m = sqrt(h1.*P./(k.*Ac));
        S = (tHETA./tHETAb)==((cosh(m*(L-x)))+(h1./(m*k))*(sinh(m*(L-x))))...
            /((cosh(m*L))+(h1./(m*k))*(sinh(m*L)));
        hT41(i-((2*tTL1)/3)) = vpasolve(S,h1,20);
        % solve for other parameters using an individual convection
        % coefficient (h)
        h = hT41(i-((2*tTL1)/3));
        % Find m and M
        m = sqrt(h.*P./(k.*Ac));
        M = sqrt(h.*P.*k.*Ac).*(tHETAb);
        % Find fin heat transfer rate (qF) and tabulate values at
        % thermocouple for plotting
                                                        qFT41(i-((2*tTL1)/3))          =
(M.*(sinh(m.*L)+(h./(m.*k))*cosh(m.*L)))/(cosh(m.*L)+(h./(m.*k))*sinh(m.*L));
        qF = qFT41(i-((2*tTL1)/3));
        % Evaluate Fin Effectiveness (epsilon)
        fINeFFECTt41(i-((2*tTL1)/3)) = qF./(h.*Ac.*tHETAb);
        % Evaluate Fin Efficiency
        fINeFFICt41(i-((2*tTL1)/3)) = qF./(h.*Af.*tHETAb);
        fINeFFIC = fINeFFICt41(i-((2*tTL1)/3));
        % Evaluate total heat transfer rate (qT)
        qTt41(i-((2*tTL1)/3)) = h.*At.*(1-(((N.*Af)/At)*(1-fINeFFIC)))*tHETAb;
```

```
        qT = qTt41(i-((2*tTL1)/3));
        % Evaluating Overall Efficiency
        oVReFFICt41(i-((2*tTL1)/3)) = qT./(h.*At.*(tHETAb));
    end
end

%% Plotting

% Temperature Plot
figure (1)
hold on
t = 1:length(T1_fo);
t1 = 1:length(T1_fr);
plot(t, T1_fo, t, T2_fo, t, T3_fo, t, T4_fo,'LineWidth', 2); % forced convection
plot(t1, T1_fr, t1, T2_fr, t1, T3_fr, t1, T4_fr,'LineWidth', 2); % free convection
title('Temperature vs Time for Forced and Free Convection');
xlabel('Time (seconds)');
s = ['Temperature (' char(176) 'C)'];
ylabel(s);
legend('Thermo1 T--Forced','Thermo2 T--Forced','Thermo3 T--Forced',...
    'Thermo4 T--Forced','Thermo1 T--Free','Thermo2 T--Free','Thermo3 T--Free',...
    'Thermo4 T--Free','location','east');
%xlim([300,1200]); % limit disregards readings below 300 seconds

% Convection Coefficient Plot
figure(2)
hold on
plot(t, hT2, t, hT3, t, hT4,'LineWidth', 2); % forced convection
plot(t1, hT21, t1, hT31, t1, hT41,'LineWidth', 2); % free convection
title('Convection Coefficient vs Time for Forced and Free Convection');
xlabel('Time (seconds)');
ylabel('Convection Coefficient, h [W/(m^2*K]');
legend('Thermo2 h--Forced','Thermo3 h--Forced','Thermo4 h--Forced'...
    ,'Thermo2 h--Free', 'Thermo3 h--Free', 'Thermo4 h--Free','location','east');
xlim([300,1200]);

% Fin Heat Transfer Rate Plot
figure(3)
hold on
plot(t, qFT2, t, qFT3, t, qFT4,'LineWidth', 2); % forced convection
plot(t1, qFT21, t1, qFT31, t1, qFT41,'LineWidth', 2); % free convection
title('Fin Heat Transfer Rate vs Time for Forced and Free Convection');
xlabel('Time (seconds)');
```

```
ylabel('Fin Heat Transfer Rate, q_{f} [W]');
legend('Thermo2 q_{f}--Forced','Thermo3 q_{f}--Forced','Thermo4 q_{f}--Forced'...
    ,'Thermo2 q_{f}--Free', 'Thermo3 q_{f}--Free', 'Thermo4 q_{f}--Free','location','east');
xlim([300,1200]);

% Fin Effectiveness Plot
figure(4)
hold on
plot(t, fINeFFECTt2, t, fINeFFECTt3, t, fINeFFECTt4,'LineWidth', 2); % forced convection
plot(t1, fINeFFECTt21, t1, fINeFFECTt31, t1, fINeFFECTt41,'LineWidth', 2); % free convection
title('Fin Effectiveness vs Time for Forced and Free Convection');
xlabel('Time (seconds)');
ylabel(['Fin Effectiveness, ' char(949) '_{f}']);
legend(['Thermo2 ' char(949) '_{f}--Forced'],['Thermo3 ' char(949) '_{f}--Forced'],['Thermo4 ' char(949)
'_{f}--Forced']...
        ,['Thermo2 ' char(949) '_{f}--Free'],['Thermo3 ' char(949) '_{f}--Free'],['Thermo4 ' char(949)
'_{f}--Free'],'location','southeast');
xlim([300,1200]);

% Fin Efficiency Plot
figure(5)
hold on
plot(t, fINeFFICt2, t, fINeFFICt3, t, fINeFFICt4,'LineWidth', 2); % forced convection
plot(t1, fINeFFICt21, t1, fINeFFICt31, t1, fINeFFICt41,'LineWidth', 2); % free convection
title('Fin Efficiency vs Time for Forced and Free Convection');
xlabel('Time (seconds)');
ylabel('Fin Efficiency, \eta_{f}');
legend('Thermo2 \eta_{f}--Forced','Thermo3 \eta_{f}--Forced','Thermo4 \eta_{f}--Forced'...
    ,'Thermo2 \eta_{f}--Free','Thermo3 \eta_{f}--Free','Thermo4 \eta_{f}--Free','location','southeast');
xlim([300,1200]);

% Total Heat Transfer Rate of Heat Sink Plot
figure(6)
hold on
plot(t, qTt2, t, qTt3, t, qTt4,'LineWidth', 2); % forced convection
plot(t1, qTt21, t1, qTt31, t1, qTt41,'LineWidth', 2); % free convection
title('Total Heat Transfer Rate vs Time for Forced and Free Convection');
xlabel('Time (seconds)');
ylabel('Total Heat Transfer Rate, q_{t} [W]');
legend('Thermo2 q_{t}--Forced','Thermo3 q_{t}--Forced','Thermo4 q_{t}--Forced'...
    ,'Thermo2 q_{t}--Free','Thermo3 q_{t}--Free','Thermo4 q_{t}--Free','location','southeast');
xlim([300,1200]);
```

```
% Overall Efficiency Plot
figure(7)
hold on
plot(t, oVReFFICt2, t, oVReFFICt3, t, oVReFFICt4, 'LineWidth', 2); % forced convection
plot(t1, oVReFFICt21, t1, oVReFFICt31, t1, oVReFFICt41,'LineWidth', 2); % free convection
title('Overall Efficiency vs Time for Forced and Free Convection');
xlabel('Time (seconds)');
ylabel('Overall Efficiency, \eta_{o}');
legend('Thermo2 \eta_{o}--Forced','Thermo3 \eta_{o}--Forced','Thermo4 \eta_{o}--Forced'...
    ,'Thermo2 \eta_{o}--Free','Thermo3 \eta_{o}--Free','Thermo4 \eta_{o}--Free','location','southeast');
xlim([300,1200]);

% End of convection code
```